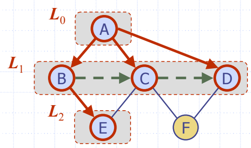
# Breadth-First Search






# Outline and Reading

- Breadth-first search (§6.3.3)
  - Algorithm
  - Example
  - Properties
  - Analysis
  - Applications
- DFS vs. BFS (§6.3.3)
  - Comparison of applications
  - Comparison of edge labels



# Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- BFS on a graph with $n$ vertices and $m$ edges takes $O(n + m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one



# BFS Algorithm

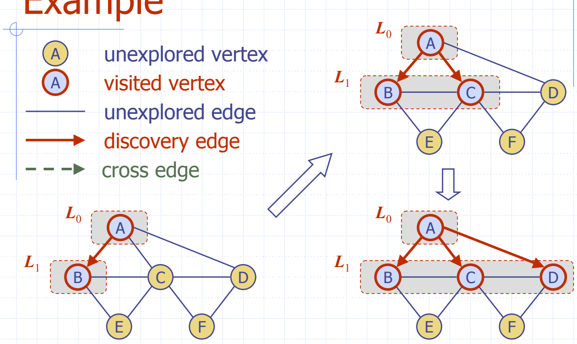
- The algorithm uses a mechanism for setting and getting "labels" of vertices and edges

```
Algorithm BFS(G)
  Input graph G
  Output labeling of the edges
     and partition of the
     vertices of G
  for all u ∈ G.vertices()
    setLabel(u, UNEXPLORED)
  for all e ∈ G.edges()
    setLabel(e, UNEXPLORED)
  for all v ∈ G.vertices()
    if getLabel(v) = UNEXPLORED
      BFS(G, v)
```

```
Algorithm BFS(G, s)
  L_0 ← new empty sequence
  L_0.insertLast(s)
  setLabel(s, VISITED)
  i ← 0
  while ¬L_i.isEmpty()
    L_{i+1} ← new empty sequence
    for all v ∈ L_i.elements()
      for all e ∈ G.incidentEdges(v)
        if getLabel(e) = UNEXPLORED
          w ← opposite(v,e)
          if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            setLabel(w, VISITED)
            L_{i+1}.insertLast(w)
          else
            setLabel(e, CROSS)
    i ← i +1
```

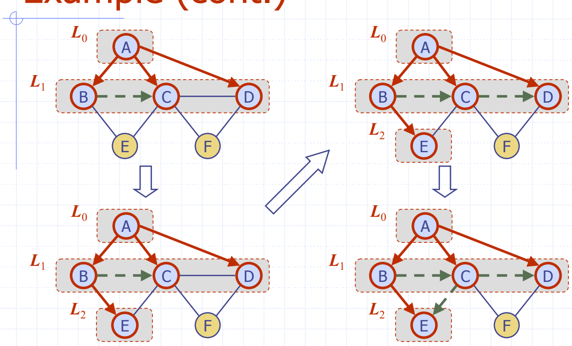


# Example

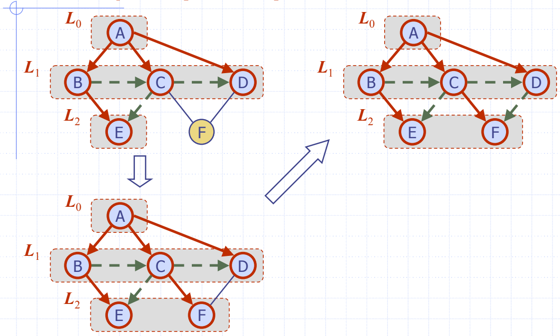





# Example (cont.)

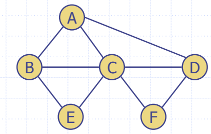


# Example (cont.)






# Properties




Notation

$G_s$: connected component of $s$

Property 1

$BFS(G, s)$ visits all the vertices and edges of $G_s$

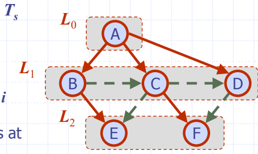
Property 2

The discovery edges labeled by $BFS(G, s)$ form a spanning tree $T_s$ of $G_s$

Property 3

For each vertex $v$ in $L_i$

- The path of $T_s$ from $s$ to $v$ has $i$ edges
- Every path from $s$ to $v$ in $G_s$ has at least $i$ edges






# Analysis

- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence $L_i$
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$



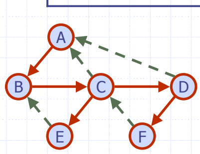
# Applications

- Using the template method pattern, we can specialize the BFS traversal of a graph $G$ to solve the following problems in $O(n + m)$ time
  - Compute the connected components of $G$
  - Compute a spanning forest of $G$
  - Find a simple cycle in $G$, or report that $G$ is a forest
  - Given two vertices of $G$, find a path in $G$ between them with the minimum number of edges, or report that no such path exists

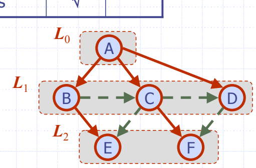


# DFS vs. BFS

| Applications | DFS | BFS |
|---|---|---|
| Spanning forest, connected components, paths, cycles | √ | √ |
| Shortest paths | | √ |
| Biconnected components | √ | |




DFS




BFS



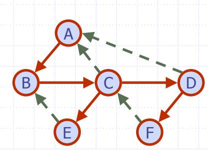
# DFS vs. BFS (cont.)

Back edge $(v,w)$

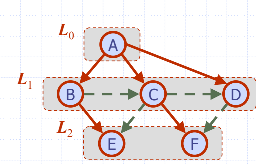
- $w$ is an ancestor of $v$ in the tree of discovery edges

Cross edge $(v,w)$

- $w$ is in the same level as $v$ or in the next level in the tree of discovery edges




DFS




BFS